GCE

Mathematics A

H230/02: Pure Mathematics and Mechanics

Advanced Subsidiary GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

Subject-specific Marking Instructions for AS Level Mathematics A

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value is not given in the paper accept any answer that agrees with the correct value to **3 s.f**. unless the question specifically asks for another level of accuracy.

Follow through should be used so that only one mark is lost for each distinct accuracy error.

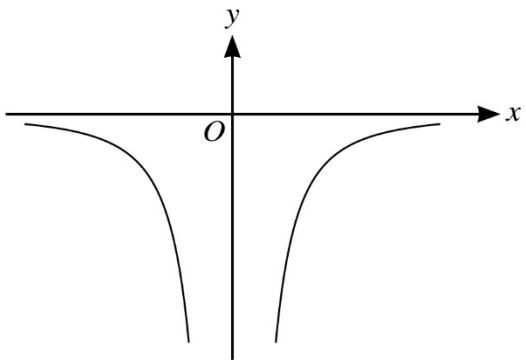
g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1			**DR** $x=\dfrac{24}{3-\sqrt{5}}=\dfrac{24\left(3+\sqrt{5}\right)}{\left(3-\sqrt{5}\right)\left(3+\sqrt{5}\right)}$	**M1**	**1.1**	Multiplying numerator and denominator by $3+\sqrt{5}$ or $-3-\sqrt{5}$	**Alternative: M1** Correct method to solve simultaneous equations formed from equating expressions to $a+b\sqrt{5}$
			$=\dfrac{24\left(3+\sqrt{5}\right)}{9-3\sqrt{5}+3\sqrt{5}-5}=\dfrac{24\left(3+\sqrt{5}\right)}{4}$	**A1**	**1.1**	Correct simplified denominator	**A1** Either *a* or *b* correct
			$=18+6\sqrt{5}$	**A1** **[3]**	**1.1**	Final answer **cao** , therefore final answer of only $6\left(3+\sqrt{5}\right)$ is **A0**	**A1** Both correct
2	**(a)**		$5\left[x^2-4x\right]+3$			No marks until attempt to complete the square	
			$=5\left[\left(x-2\right)^2-4\right]+3$ $\quad p=5$	**B1**	**1.1**	Must be of the form $5\left(x\pm\alpha\right)^2\pm\cdots$	
			$\left(x-2\right)^2$	**B1**	**1.1**		
			$=5\left(x-2\right)^2-17$ $\quad r=-17$	**B1** **[3]**	**1.1**		
2	**(b)**		Minimum point $\left(2,-17\right)$	**B1ft** **B1ft** **[2]**	**1.1** **1.1**	Follow through their $-q$ Follow through their *r*	Or by differentiation
2	**(c)**		$x=2$	**B1ft** **[1]**	**1.1**	Follow through their *x* coordinate in part (b)	

Question			Answer	Marks	AO	Guidance	
3	(a)			B1 [1]	1.1	Curve in both quadrants: • Correct shape, symmetrical, not touching axis • Asymptote the axes • Not finite • Allow slight movement away from asymptote at one end but not more	**N.B. Ignore 'feathering' now that answers are scanned.**
3	(b)		$y = -\frac{1}{(x-2)^2}$	M1 A1 [2]	1.1 2.5	$(y=)-\frac{1}{(x-2)^2}$ or $(y=)-\frac{1}{(x+2)^2}$ Fully correct, must include '$y=$'	
3	(c)		$\left(\frac{1}{2}, -2\right)$	B2 [2]	1.1 1.1	B1 for each coordinate	

Question			Answer	Marks	AO	Guidance	
4	(a)		$(2-5x)^5 = 2^5 + {}^5C_1 2^4(-5x) + {}^5C_2 2^3(-5x)^2 + \ldots$	M1	1.1a	Attempt at least 2 terms – products of binomial coefficients and correct powers of 2 and $-5x$	Allow $\pm 5x$ – allow expansion of $\left(1 \pm \frac{5}{2}x\right)^5$
			$32 - 400x$	A1	1.1		
			$+2000x^2$	A1	1.1		Do not allow from $+5x$
				[3]			
4	(b)		$\left(1 + 2ax + a^2x^2\right)\left(32 - 400x + 2000x^2 + \ldots\right)$	M1*	2.1	Expand first bracket, multiply by part (a) to obtain the two relevant terms in x	Ignore terms in x^2
			$64a - 400 = 48 \Rightarrow a = \ldots$	Dep*M1	1.1	Equate sum of the two relevant terms to 48 and attempt to solve for a	**M1 only** for $2a - 400 = 48$ (oe e.g. with consistent x)
			$a = 7$	A1	2.2a	Obtain $a = 7$ only	
				[3]			
5	(a)		$k = 3$	B1	1.1		
				[1]			
5	(b)		$(1-4)^2 + (2-k)^2 = 13$	M1	1.1a	oe e.g. allow consistent use of square roots – must be using subtraction in brackets	May be implied by one correct value for k
			$k = 0$	A1	1.1		
			$k = 4$	A1	1.1		
				[3]			
5	(c)		$\frac{4-2}{7-1} = \frac{k-5}{4-3}$ oe or $\frac{5-2}{3-1} = \frac{4-k}{7-4}$ oe	M1	3.1a	or $\frac{5-4}{3-7} = \frac{k-2}{4-1}$ oe – must be consistent application of gradients (allow one sign error)	Any one of these three solutions
			$k = \frac{16}{3}$ or $k = -\frac{1}{2}$	A1	1.1	$k = \frac{5}{4}$	
				[2]			
6	(a)		**DR**				

Question			Answer	Marks	AO	Guidance	
			$6\left(1-\sin^2\theta\right)=\frac{\sin\theta}{\cos\theta}\left(\cos\theta\right)+4$	**M1**	**3.1a**	Correct use of both $\cos^2\theta=1-\sin^2\theta$ and $\tan\theta=\frac{\sin\theta}{\cos\theta}$	
			$6-6\sin^2\theta=\sin\theta+4 \Rightarrow 6\sin^2\theta+\sin\theta-2=0$	**A1**	**2.1**	**AG**	Must show sufficient working to justify the given answer
				[2]			
6	**(b)**		**DR** $\left(2\sin\theta-1\right)\left(3\sin\theta+2\right)$	**M1**	**1.1a**	Attempt to solve 3-term quadratic	Ignore incorrect use of inequalities for first three marks
			Critical values occur when $\sin\theta=\frac{1}{2}$ and $\sin\theta=-\frac{2}{3}$	**B1**	**1.1**		
			Critical values are $\theta=30,150,222,318$	**B1**	**1.1**	Any three correct critical values	221.8103… 318.1896…
			$0<\theta<30$ or $150<\theta<222$ or $318<\theta<360$	**B1**	**1.1**	B1 for one correct interval – 3sf or better (condone use of x)	Condone $\leq$ oe
				A1	**2.5**	Cao (all three intervals) – 3 sf or better For those that have $\sin\theta=-\frac{1}{2}$ and $\sin\theta=\frac{2}{3}$ can score M1 (if **DR** seen) then **SC B1** for one 'correct' interval (condone $\leq$ oe) or **SC B2** for all three 'correct' intervals which are $\theta<42, 138<\theta<210, \theta>330$ (so max. 3/5)	Allow $\theta<30$, $150<\theta<222$, $318<\theta$
				[5]			
7			$\frac{32}{3}$	**B1**	**1.1**	Seen or implied by later working	

Question			Answer	Marks	AO	Guidance	
				M1*	**3.1a**	Attempt integration on a 3 term quadratic in x	(increase in power by 1 for at least 1 term but not just multiplying each term by x)
			$\int(-x^2+6x-5)\,\mathrm{d}x=-\frac{x^3}{3}+3x^2-5x$	**A1**	**1.1**	Ignore lack of $+c$	
			$-\frac{a^3}{3}+3a^2-5a-\left(-\frac{5^3}{3}+75-25\right)$	**Dep*M1**	**1.1**	$\pm\left(\mathrm{F}(a)-\mathrm{F}(5)\right)$	
			$\frac{32}{3}+\frac{a^3}{3}-3a^2+5a+\frac{25}{3}=19$	**A1**	**1.1**	oe	
			$a^3-9a^2+15a=0\Rightarrow a^2-9a+15=0\because a\neq 0$	**M1**	**3.1a**	solve their cubic (which comes from attempt at both areas and 19) leading to an exact value for a	**Dependent on both previous M marks**
			$a\neq\frac{9-\sqrt{21}}{2}\because a>5$	**B1**	**3.2a**	**BC –** must give a reason for rejection of this value of a	Allow rejection of 2.21
			$a=\frac{9+\sqrt{21}}{2}$ only	**A1**	**2.2a**	**BC**	
				[8]			
8	**(a)**		$\log_2 x^2\left(=\log_2\left(kx-1\right)+3\right)$	**B1**	**1.2**	Using $a\log b=\log\left(b^a\right)$	

Question			Answer	Marks	AO	Guidance	
			$\log_2\left(\frac{x^2}{kx-1}\right)=3$	**M1***	**2.1**	Re-arranging and correctly combining both log terms	Or re-write 3 as $\log_2 8$ and then combining e.g. $2\log_2 x$ $=\log_2\left(8(kx-1)\right)$
			$\frac{x^2}{kx-1}=2^3$ $x^2=8(kx-1)$	**Dep*M1**	**1.1**	Correctly remove logs	$x^2=8(kx-1)$
			$x^2-8kx+8=0$	**A1**	**1.1**	**AG**	Must show sufficient working to justify the given answer (i.e. at least one more line of working from previous M mark)
				[4]			
8	**(b)**		$b^2-4ac=0\Rightarrow(-8k)^2-4(1)(8)=0$	**M1**	**3.1a**	Use of $b^2-4ac=0$	Or state equation must be of the form $(x+p)^2=0$
			$k=(\pm)\frac{1}{\sqrt{2}}$	**A1**	**1.1**	oe exact	with $p^2=8$
			$k=\frac{1}{\sqrt{2}}\Rightarrow x=2\sqrt{2}$	**A1**	**2.2a**	**BC** oe exact	so $x=(\pm)2\sqrt{2}$
			$k=-\frac{1}{\sqrt{2}}\Rightarrow x=-2\sqrt{2}$ and as $\log_2 x$ is only defined for $x>0$ so $x\neq-2\sqrt{2}$	**A1**	**3.2b**	**BC** oe statement for rejection of negative value of x (allow decimal argument)	reject $x=-2\sqrt{2}$ with valid reason
				[4]			

Question			Answer	Marks	AO	Guidance	
9			$\mathbf{F} = \begin{pmatrix} -9 \\ 1 \end{pmatrix}$	**B2** **[2]**	**1.1** **1.1**	B1 for one correct value	Allow **–9i + j** **SC B1** for $(-9,1)$ or $(-9 \quad 1)$
10	(a)		$v = pt^2 + qt + r$ $t = 0, v = 18 \Rightarrow r = 18$ $t = 5, v = 9 \Rightarrow 25p + 5q + 18 = 9$ $\frac{dv}{dt} = 2pt + q$ $t = 5, \frac{dv}{dt} = 0 \Rightarrow 10p + q = 0$ $p = \frac{9}{25}, q = -\frac{18}{5}$	**B1** **M1** **B1** **M1** **A1** **[5]**	**3.4** **1.1** **3.1b** **1.1** **1.1**	Substitutes $t = 5, v = 9$ into quadratic Substitutes $t = 5$ and sets $\frac{dv}{dt} = 0$ **BC** (oe e.g. exact decimals)	Allow with r Dependent on one term differentiated correctly
10	(b)		$\int_2^5 \left(\frac{9}{25}t^2 - \frac{18}{5}t + 18\right) dt$ $+ 9 \times 5$ = 75.24 m	**M1** **B1** **A1** **[3]**	**3.4** **1.1** **1.1**	Using their values of p, q and r in an attempt to find the distance travelled from 2 to 5 by integration For distance travelled from 5 to 10 **BC** cao (oe)	

Question			Answer	Marks	AO	Guidance	
11	(a)			M1*	3.3	Attempt N2L for P or Q	Must include correct number of terms – use of weight for mass is **M0**
			For P: $40 - T - 8 = 3a$	A1	1.1		
			For Q: $T - 2g = 2a$	A1	1.1		
			$32 - 2g = 5a \Rightarrow a = ...$	Dep*M1	1.1	Attempt to solve simultaneous equations	
			$a = 2.48\ \text{m s}^{-2}$	A1	2.2a	**AG**	Must show sufficient working to justify the given answer
						M1 A2 for $40 - 8 - 2g = 5a$ for **M1** must have correct number of terms and mass must be 5 not $5g$	
				[5]			
11	(b)		$T - 2g = 2(2.48)$	M1	3.4	Substitute given value of a into either equation	Must include correct number of terms – use of weight for mass is **M0**
			$T = 24.56\,\text{N}$	A1	1.1	cao	Allow 24.6
				[2]			
11	(c)		$v = 2.48(0.5)$	B1	3.4	Speed after 0.5 seconds	1.24
			$s = 0.5(2.48)(0.5)^2$	B1	3.4	Distance travelled in this time	0.31
				M1	3.1b	Applying $s = ut + 0.5at^2$ correctly — allow sign errors	**M0** if not using relevant displacement
			$-(2 + 0.31) = 1.24t - 0.5(9.8)t^2$	A1	1.1		
			$t = 0.825\,\text{s}$	A1	2.2a	**BC**	0.8246986…

Question			Answer	Marks	AO	Guidance	
11	(c)		**ALT** $v = 2.48(0.5)$ $s = 0.5(2.48)(0.5)^2$ $0 = 1.24^2 - 2(9.8)s \ (\Rightarrow s = 0.0784)$ $2 + 0.31 + 0.0784 = 0.5(9.8)t_1^2$ $t_1 = 0.698$ $t = 0.825$	**B1** **B1** **M1** **A1** **A1** **[5]**		Speed after 0.5 seconds Distance travelled in this time Complete method to calculate the time down Correct value for time down	1.24 0.31
11	(d)		$v^2 = 1.24^2 + 2(-9.8)(-2.31)$ $v = 6.84 \text{ m s}^{-1}$	**M1** **A1** **[2]**	**3.3** **1.1**	Applying $v^2 = u^2 + 2as$ correctly with their 1.24 and 2.31 **or** any other complete method Allow 6.85	**M0** if not using total time or relevant displacement 6.8420464…
11	(e)		19.6 N	**B1** **[1]**	**3.4**	Accept 2*g*	
11	(f)		e.g. include a more accurate value for *g* e.g. include a variable resistance in the model rather than a constant e.g. include the dimension of the pulley in the model so that the string is not parallel to the table e.g. include a frictional force at the pulley	**B1** **[1]**	**3.5c**		

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